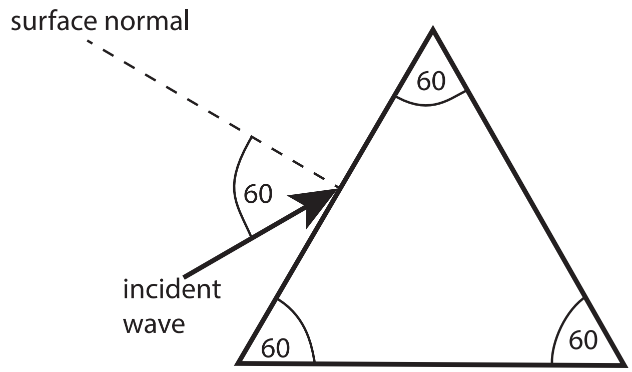
Waves and Optics, Exam, 23/01/2020

4 questions, 42 points total

Write your name and student number on each answer sheet. Use of a calculator is allowed.

1. (12 points)

Sapphire has a refracive index of about 1.77. We are going to study the reflective and refractive properties of this material. A triangular prism (with a triangular base and rectangular sides) is made from sapphire, and a plane wave light field is coming in from air (n=1) under an incident angle of 60 degrees with respect to the surface normal. The situation is sketched as a top view of the prism in the figure on the right.



(a) (4 points) Use Snell's law ($n_i \sin(\theta_i) = n_t \sin(\theta_t)$) to calculate the angle of the light beam as it *leaves* the prism, relative to the normal of the surface where the light beam leaves the prism.

(b) (2 points) Suppose the incoming wave has equal components of s and p polarization. What can you say about the ratio of these two components in the light that is reflected light from the surface of the prism? Explain the responsible mechanism.

(c) (2 points) As we all know, a prism can be used to disperse the wavelength components present in for example white light. However, in the previous subquestion there was no mention of the influence of the wavelength of the light. What is missing from Snell's law, as given above? How is this effect taken into account in the model that describes the interaction of the light with the atoms in the material?

(d) (4 points) For a light beam coming from within the sapphire, approaching the interface with air, calculate the critical angle for total internal reflection. How high would the refractive index at least have to be for the light to keep bouncing around inside the prism?

Answer model:
a) For the first interface we find $\frac{n_i}{n_t}\sin(60) = \sin(\theta_t)$, *which gives* $\theta_t = 29.3$ *degrees. This light beam propagates to the second interface, where we find from the geomtry that it has an angle of 30.7 degrees with respect to the surface normal. Upon leaving the prism we find the angle to be 64.6 degrees with respect to the surface normal. (+1 point for correct* θ_t *w.r.t. first interface, +1 point for correct* θ_i *w.r.t. second interface, +2 points for correct* θ_t *w.r.t. second interface)*
b) The incoming angle is pretty close to the Brewster's angle, which has a minimum for p-polarisation. Therefore, the reflected light mostly consists of s-polarized light. (+1 point for

saying $E^{(s)} > E^{(p)}$, +1 point for explaining that the dipole oscillation vector is related)
c) The index of refraction is actually wavelength dependent. That is not specified in Snell's law as given in question a). The Lorentz model describes the interaction of the electromagnetic wave with the atoms in the dielectric material, where the atoms are polarized by the light field. The electrons are treated as being connected with springs to the atomns, with the springs having a restoring force, and also a resonance frequency. Around the resonance frequency the index of refraction depends on the wavelength. (+1 point for saying that, generally, the refractive index depends on ω, +1 point for relating to the resonance behaviour of a spring system)
d) Snell's law:

$$\begin{aligned} n_i \sin\theta_i &= n_t \sin\theta_t \\ \frac{n_i}{n_t}\sin\theta_i &= \sin\theta_t \end{aligned}$$

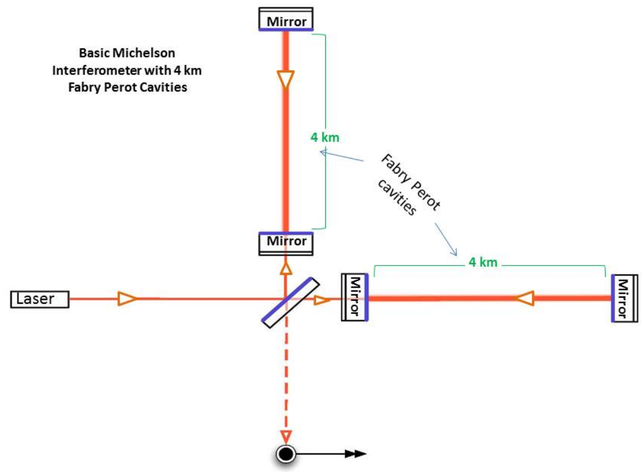
$\theta_i = \theta_c$ *when the left hand side equals 1 (when the left side exceeds 1, θ_t becomes complex):*

$$\begin{aligned} \frac{n_i}{n_t}\sin\theta_c &= 1 \\ \theta_c &= \arcsin\frac{n_t}{n_i} \\ &= \arcsin\frac{1}{1.77} \\ &= 0.60 rad = 34.4\,\mathrm{deg} \end{aligned}$$

If the critical angle was 30 degrees or smaller, which would happen for a refractive index of 2 or more, the light would keep bouncing around inside the prism. (+2 points for the correct calculation of θ_c (or +1 for realising θ_t breaks), +1 point for saying that $\theta_c \leq 30\,\mathrm{deg}$, +1 point for saying $n \geq 2$)

2. (8 points)

A simplified version of the optical layout of the LIGO gravitational wave detector is shown in the figure. It basically is a Michelson interferometer with Fabry-Perot cavities in the two arms.



(a) (2 points) Why are the Fabry-Perot cavities used in this setup?

(b) (2 points) In the original experiment to detect the motion of the earth through ether, Michelson and Morley used a light source with a broad frequency spectrum. Explain how it is possible to observe interference using a broadband light source, even though the coherence length of broadband light is much smaller than the size of the setup.

(c) (2 points) In the gravitational wave detector narrow-band laser light is used. Is it possible to observe good fringe visibility if the two arms of the interferometer are somewhat different in length? How can the coherence length of the laser light be used to quantify this effect?

(d) (2 points) The Fabry-Perot cavities have to be stable, which means that the radii of curvature (R_1 and R_2) and the distance between the mirrors (L) have to be such that the laser light bounces around many times in the cavity. A stable cavity is formed if

$$0 < (1 - \frac{L}{R_1})(1 - \frac{L}{R_2}) < 1.$$

The distance between the mirrors is 4 km. If one mirror is flat, what are the requirements for the radius of curvature of the other mirror to form a stable cavity?

Answer model:
a) They are used to increase the effective length of the two arms of the Michelson interferometer. This in turn increases the sensitivity for the detection of gravitational waves. (+2 points for correct explanation)
b) This is only possible if the length of the two arms is exactly the same. Also, the light in both arms has to dispersed by exactly the same amount. (+2 points for explaining the arms must be identical)
c) Yes, that is possible, because the coherence length for a narrow-band laser is sufficiently long that interference also occurs when the two interfering waves are displaced by (many) multiples of the wavelength. The coherence length of the laser light quantifies how much one of the arms can be different from the other, before losing fringe visibility. (+1 point for noting that the

lengths can be somewhat different, +1 point for noting that coherence length quantifies how different)
d) $L = 4000(0.5pt), R_1 = \infty$ *(0.5 pt).* $0 < 1 - 4000/R_2 < 1$ *(0.5 pt) which is fulfilled if* $R_2 > 4000$ *(0.5 pt).*

3. (8 points) An object with a height of 10 cm is imaged by a thin lens, placed at a distance of 60 cm from the object, with a focal length of 20 cm, onto a screen.
   (a) (4 points) At what distance of the lens does the screen have to be placed, to see the sharp image? You can either use a sketch with a few key rays to determine this, or use the lens makers forrmula.
   (b) (2 points) What is the magnification of the formed image?
   (c) (2 points) The lens that is used has a anti-reflection coating, with a quarter wavelength thickness. Explain how such a coating decreases the reflectivity.

Answer model:
a) With a sketch like figure 9.14, we find that the screen is to be placed at 30 cm. (2pts figure, 2pts correct distance) From the lens makers formula, $1/60 + 1/x = 1/20$*, so* $x = 1/(1/20 - 1/60) = 30$ *(2pt correct formula, 2pt correct answer).*

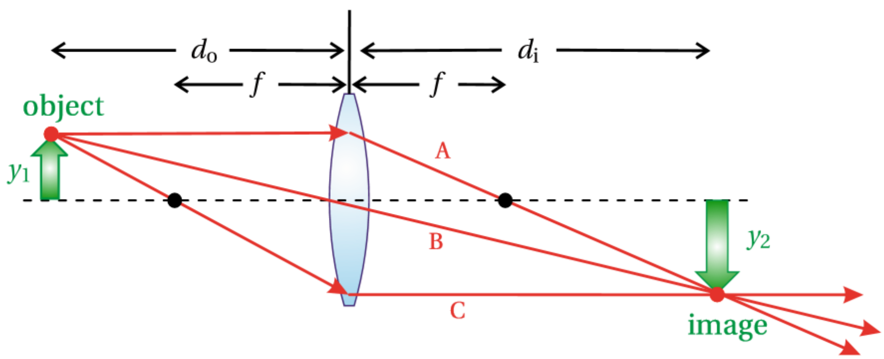


Figure 9.14 Formation of a real image by a thin lens.

b) The image is inverted, so the magnification is negative. For the given distance, we find a magnification of -1/2. (1pt correct formula, 1pt correct answer)
c) With a quarter wave thickness, the reflections from the front and back side of the coating are π *out of phase, and therefore cancel each other (1pt). The result is reduced reflection, and increased transmission(1pt)* OR
$d = \frac{\lambda}{4}$ *minimises the* $\sin^2(\Phi)$ *in equation 4.15 (1pt). Thereby maximising transmittance. (1pt)*

4. (14 points) A student is given a red laser ($\lambda = 633$ nm) and a green laser ($\lambda = 532$ nm) (that both produce a collimated laser beam of 5 mm diameter), a series of slides with various transmission patterns (labelled with their shape and size), a few lenses and an imaging screen. Of course, she also uses safety glasses in her experiments.
   (a) (2 points) She wants to see significant diffraction from a circular aperture. Which aperture radius is better to use, 1 mm or 1 μm? Why?

(b) (2 points) The next objective is to make sure that the diffraction pattern she sees is the Fraunhofer diffraction pattern. How can she do this with the tools she is given? Make a schematic drawing, indicating the relevant distances.

(c) (2 points) Now she studies the difference between the Fraunhofer diffraction pattern for the red and the green laser light, using a slide with 5 parallel slits on it. Qualitatively, how (besides the color) are the two patterns different?

(d) (4 points) She also finds a slide that has a pattern of a very large number of parallel slits on it. The pattern is 2x2 cm in size. She decides to build a monochromator that can discriminate between the red light and the green light. For this she also gets a moveable slit and a photodiode, to detect the light. Draw a setup that would work.

(e) (4 points) One of the slides has an image of her teacher on it. For some reason he is wearing a large hat with a print of vertical stripes on it. The task given to her is to devise a setup where this image is projected on the screen, but with the pattern on the hat filtered out. What material does she need, and how would she use it to achieve this goal?

Answer model:
a) 1 μm, because the diffraction pattern will be much larger. This is because the aperture size is closer to the wavelength. (+1 point correct aperture, +1 point correct reasoning)
b) She has to use a lens, and place the screen at the focal distance of the lens. The distance between the aperture and lens is not too important, as long as it is at least the focal length. (+1 point correct setup, +1 point correct distances. (Only saying 'a large distance' (without lens) does not guarantee Fraunhofer diffraction. Unless mentioned that the screen needs to be moved until the pattern stops changing))
c) The spacing between the interference fringes is different for the two colors. The central interference fringe is at the same spot, but the first order maximum is further away for the longer wavelength. (+2 points correct explanation)
d) The simplest version: laser - slide - lens - movable slit at focus distance - detector. However, better is to focus the laser beam down through a pinhole, and then use another lens with a larger focal length to make the beam larger, so that all slits are being used for the interference pattern. (+4 points fully correct layout, -1 point using additional equipment)
e) She should put a mask in the Fourier plane, to block out the dots that are produced by the vertical stripes. The vertical stripes will produce spots in the middle of the fourier plane, horizontally displaced. So the mask could be a a slide that blocks a horizontal line at the middle of the fourier plane. A second lens should be used to image this fourier plane onto the screen. (+1 point mentioning dots, +1 point correct mask in fourier plane, +2 points correct setup)